Math 3235 Probability Theory

2/23/23

$A \quad B \quad C$ independent

Show that A $B \cap C$ are indep.

$$\mathbb{P}(A \cap (B \cap C)) = \mathbb{P}(A)\,\mathbb{P}(B \cap C)$$

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$A \quad B \cup C$ are independent

$$\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}(A)\,\mathbb{P}(B \cup C)$$

$$\mathbb{P}(B \cup C) = \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B \cap C)$$

$$= \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B)\mathbb{P}(C)$$

$$\mathbb{P}(A \cap (B \cup C)) =$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}((A \cap B) \cup (A \cap C)) =$$

$$\mathbb{P}(A \cap B) + \mathbb{P}(A \cap C) - \mathbb{P}(A \cap B \cap C)$$

$$(A \cap B) \cap (A \cap C) = A \cap B \cap C$$

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if $A \perp B \iff A \perp B^c$

$A^c \perp B$

$A^c \perp B^c$

$$A \perp B \cup C$$

$$B \cup C = (B^c \cap C^c)^c$$

$A \perp B \cup C \iff A \perp (B \cup C)^c$

$$(B \cup C)^c = B^c \cap C^c$$

A, B^c, C^c are indep.

N Poisson cars arriving

Each car has prob p of requiring service.

μ

X_i is a Bernoulli r.v. p

N Poissonian par μ

$$N_s = \sum_{i=1}^{N} X_i$$

$$G_{X_i}(s) = q + ps$$

$$G_N(s) = e^{\mu(s-1)}$$

$$G_{N_S}(s) = G_N(G_{X_i}(s)) = e^{\mu(q+ps-1)} =$$
$$= e^{\mu p(s-1)}$$

Q_S is Poisson $p\mu$.

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80 bulbs working at the end of second month.

How many of the 20 bulbs broken at the end of 2nd month were working at the end of the first month?

$A = \{$ bulb broken end 2nd month $\}$

$B = \{$ bulb broken after 1 month $\}$

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} = \frac{\mathbb{P}(B)}{\mathbb{P}(A)}$$

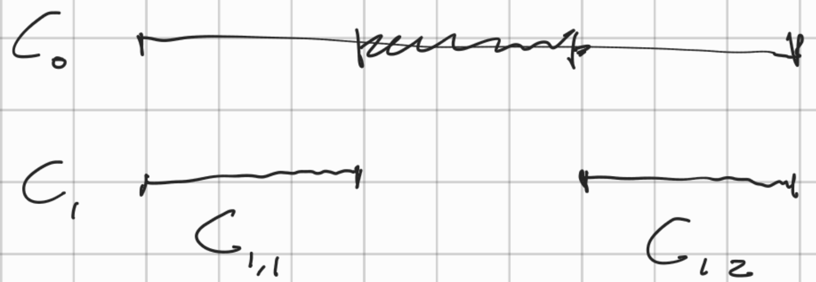
$\mathbb{P}(B) = 0.1$

$$\mathbb{P}(A) = \underbrace{\mathbb{P}(B)}_{=0.1} \underbrace{\mathbb{P}(A \mid B)}_{=1} + \underbrace{\mathbb{P}(B^c)}_{=0.9} \underbrace{\mathbb{P}(A \mid B^c)}_{=0.15}$$

$$= 0.23$$

$\mathbb{P}(B \mid A) = 0.43$

$X = 80 + Y$ $\quad Y$ is Binomial $\quad 20 \quad 0.43$.



$C_0 = [0, 1]$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

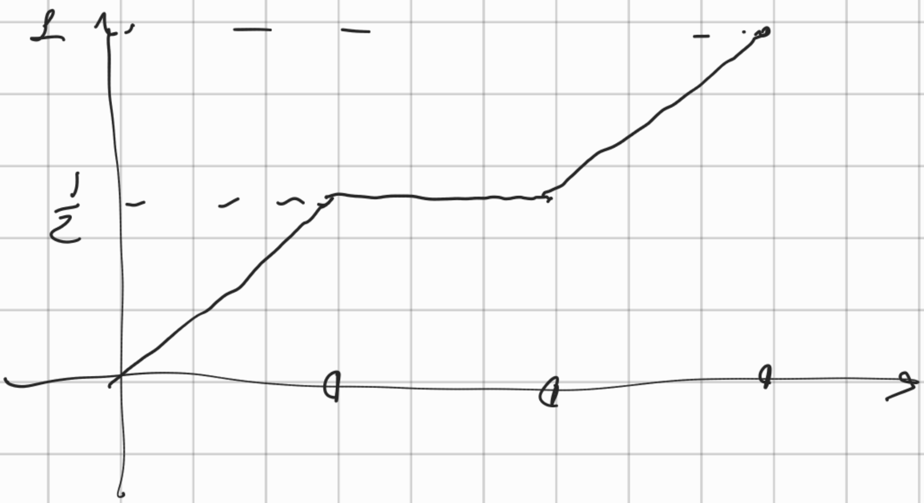
Uniform dist on $[0, \frac{1}{3}] = C_{1,1}$

$$F_{[0,\frac{1}{3}]}(x) = F(3x) = \begin{cases} 0 & x \le 0 \\ 3x & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

$$[\tfrac{2}{3}, 1] = C_{1,2}$$

$$F_{[\frac{2}{3},1]} = F(3(x - \tfrac{2}{3}))$$

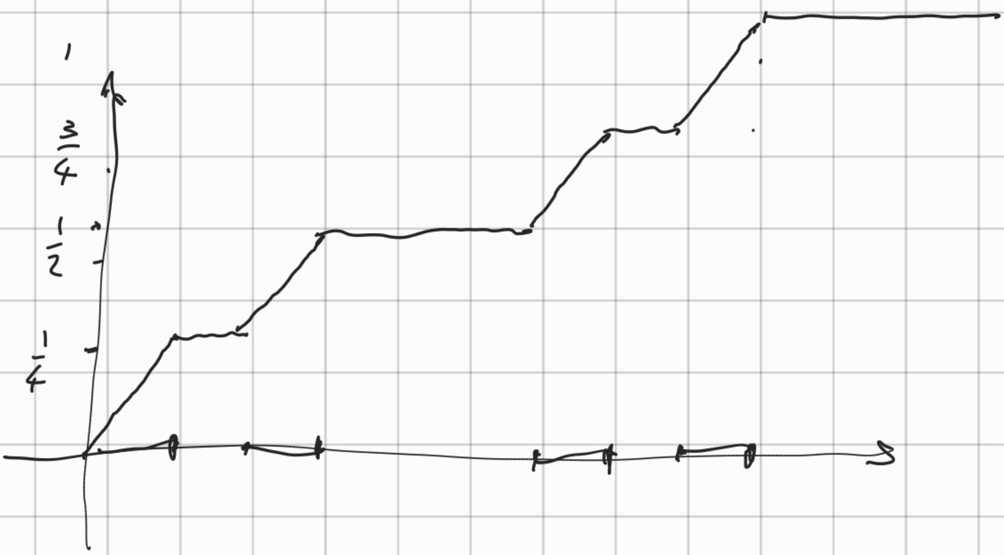
$$F_{C_1}(x) = \frac{1}{2} F_{[0,\frac{1}{3}]}(x) + \frac{1}{2} F_{[\frac{2}{3},1]}$$



$$F_{C_1}(x) = \frac{1}{2} F_{C_0}(3x) + \frac{1}{2} F_{C_0}(3(x - \tfrac{2}{3}))$$

Uniform dist. on C_2 has c.d.f.

$$F_{C_2}(x) = \frac{1}{2} F_{C_1}(3x) + \frac{1}{2} F_{C_1}\left(3x - \frac{2}{3}\right)$$



C_n n approximant to the Cantor set

C_n is formed by 2^n interval of length 3^{-n}

Total length of C_n is $\left(\frac{2}{3}\right)^n$

$$C = \bigcap_{n=0}^{\infty} C_n$$

C has total length 0.

$x \in [0,1]$ $\quad 0.\sigma_1 \sigma_2 \sigma_3 \ldots \sigma_n$

$\sigma_i \in \{0, 1, 2\}$

C is The set of x That never contain 2 in Their Ternary representation

C_1 is all x 0.1
0.3

C_2 " " " 0.11 0.13
0.31 0.33

Cantor set is not contable has Total length 0.

$$F_C(x) = \lim_{n \to \infty} F_{C_n}(x)$$

$F_C(x)$ is continuous.

$F_C(x)$ is a c.d.f.

$F_C'(x) = 0$ for almost every x.

if $F_C(x)$ were the c.d.f. of a cont. r.v. $\Longrightarrow$ $f_C(x) = 0$

$$F_C(1) - F_C(0) \neq \int_0^1 F_C'(x)\,dx$$